STRUCTURE OF THE BOUNDARY LAYER BETWEEN A RAREFIED PLASMA AND A MAGNETIC FIELD*

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The stationary problem of the structure of the plane boundary layer between a magnetic field and a collisionless plasma is solved. A method is proposed for numerically solving a system of nonlinear equations for a selfconsistent electromagnetic field. This method makes it possible to obtain, for a broad class of plasma particle distribution functions, a complete spatial picture of the boundary layer without the assumption of electrical neutrality. Results of specific calculations for normal and oblique incidence of a plasma on a "magnetic wall" are discussed. The entire study is carried out in relativistic-invariant form.

The first theoretical examination of the equilibrium transition layer associated with reflection of a plasma stream from a "magnetic wall" was made by Chapman and Ferraro [1]. In recent years, work in this direction, associated with various problems in astrophysics and plasma physics in which the plasma is screened for a finite time from an external magnetic field, has been pursued by a number of authors [2-7]. The majority of these authors assume that the plasma is strictly neutral over the entire boundary layer, which makes it possible to limit the investigation to the self-consistent magnetic field H. Then, in view of the nonlinearity of the initial equations, best results are obtained for δ-type plasma particle distribution functions (monoenergetic streams). For nonrelativistic energies, the "neutral" theory gives the order of magnitude of the characteristic dimensions of the boundary layer and in some particular problems, the form of the spatial relation for H.

From the viewpoint of the general theory, it is of great interest to obtain a complete spatial picture of the self-consistent electromagnetic field over the entire boundary layer for extended distribution functions without limiting oneself to the particle energies. Naturally, in view of the great mathematical difficulties, progress in this direction can only be made by resorting to modern computer technology.

A detailed mathematical formulation of such a problem in plane geometry for a nonrelativistic rarefied plasma is solved in relativistic-invariant form. The suitability of this rigorous approach to the problem derives from the following considerations:

1. Upon reflection of a plasma stream incident on a "magnetic wall" at the turning point a large part of the kinetic energy is transmitted to the light component. In this case, the electrons may become relativistic [9, 10, 2].

2. Consideration of the relativistic energies of plasma particles is of independent interest in certain problems involving the interaction of a plasma with a nonuniform magnetic field in outer space.

3. In the region of high plasma particle energies, the basic spatial scale factors of the problem are approached: the "magnetic" or characteristic layer thickness

$$L = \sqrt{Rr}$$

and the "electrostatic" or Debye radius D (R, r are the Larmor radii of the positively and negatively charged particles, respectively). This fact permits one to make effective use of numerical methods for finding the complete spatial picture of the boundary layer. Note also that it is particularly at high particle energies that effects associated with plasma polarization appear most clearly and the classical ideas of energy redistribution among components, associated with the "neutral" theory of the boundary layer, are most blatantly violated.

§ 1. First let us recall the formulation of the stationary problem for a plane boundary layer (see [3, 8]).

A plasma unperturbed at $x \to -\infty$ is in dynamic equilibrium with a magnetic field H_0 uniform at $x \to \infty$ (Fig. 1). The plasma is assumed to be sufficiently rarefied, i. e., the range of the particles $l_\pm \gg R$ and collisions are not taken into account. We shall assume that a stationary regime is realized, all the physical quantities depending only upon one space coordinate, x. The plasma as a whole is also neutral at $x \to -\infty$:

$$\int_{-\infty}^{\infty} (n_+(x) - n_-(x))\,dx = 0\,,$$

$$n_+(-\infty) = n_-(-\infty) = n_0\ .$$

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We assume that at $x \to -\infty$ the electric current in the yz plane is zero. (For simplicity, we will consider a two component plasma, where the charge on the individual particle $\varepsilon_- = -\varepsilon_+ = -e$. The subscripts -, + denote quantities pertaining to any two particles with negative and positive charge, respectively.) All charged particles are elastically reflected from the "magnetic wall" ($n_+(\infty) = n_-(\infty) = 0$) and intersect the plane $x = C$, $C \to -\infty$, i. e., the system under consideration does not contain particles with finite trajectories. Moreover, let $H_0 \parallel O_z$. Thus, within the framework of a one-dimensional geometry, the directions of all the vector quantities are defined, and, in particular (Fig. 1), the vector potential A(x) and the electric current j(x) screening the plasma from the magnetic field are parallel to the y-axis, the vector $\mathbf{H}(x) = \nabla \times \mathbf{A}(x)$ is parallel to the O_z-axis; the electrostatic field strength vector $\mathbf{E}(x) = -\nabla\Phi(x)$ is directed along the x-axis. This vector is associated with charge separation following from the unequal moment a of positive and negative particles, as a result of which $R \neq r$.

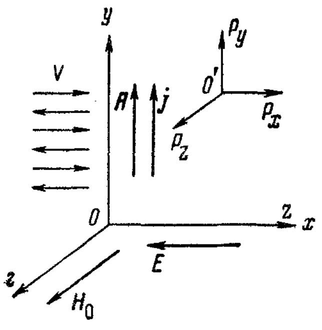

Fig. 1. Direction of principal vector quantities in the plane boundary layer.

The problem consists in finding, for the equilibrium plasma configuration described, the steady selfconsistent electromagnetic field whose specific form will clearly be determined by the particle distribution in the uniform plasma at $x \to -\infty$.

§ 2. We shall describe a system of collisionless charged particles in a selfconsistent electromagnetic field by means of a relativistic kinetic equation of the form proposed in [11]:

$$[GF] \equiv u_k \frac{\partial F}{\partial x_k} - \frac{\partial G}{\partial x_k}\frac{\partial F}{\partial p_k} = 0 \qquad \left(p_k = mcu_k + \frac{\varepsilon A_k}{c}\right). \tag{2. 1}$$

Here x_k, u_k, and p_k are, respectively, the four-dimensional radius vector, four-dimensional velocity, and generalized four-dimensional momentum of the particle, m is the particle mass, c is the speed of light, and

$$G(x, p) = -\sqrt{-[p_k - (\varepsilon/c)A_k]^2}, \tag{2. 2}$$

where $A_k = \{A_x, A_y, A_z, i\Phi\}$ is the four-dimensional field potential. Let us examine the physical meaning of the function F(x, p) in more detail.

In [11] the relativistic-invariant description of the particle distribution in coordinate and momentum space is based on a vector $F_k(x, p)$ (four-dimensional) such that

$$\varepsilon \int F_k d^4p = \langle j_k \rangle \qquad (k = 1, 2, 3, 4), \tag{2. 3}$$

where $j_k = \{j_x, j_y, j_z, ic\rho\}$ is the ordinary four-dimensional particle charge and current density vector. Since each particle makes a contribution to the current in the direction of its velocity, we have the equality $F_k = Fu_k$. In accordance with the terminology of [11], the scalar F(x, p) will be termed the scalar distribution function. The functions introduced in relativistic-invariant form enable us to describe a system of particles with different rest masses. If the latter are the same for all particles, then F contains a δ-function

$$F(x, p) = icf(x, \mathbf{p})\,\delta\left(\sqrt{-[p_k - (\varepsilon/c)A_k]^2} - mc\right). \tag{2. 4}$$

The scalar f(x, **p**), which, owing to the presence of the δ-function, may be considered as a function of only three momenta coincides with the ordinary distribution function, which is easy to show from the definition of F. Thus, a relation between F(x, p) in Eq. (2. 1) and the distribution function f(x, **v**) used in [8] is established by relation (2. 4).

With the aid of (2. 3) we may write the equations of the self-consistent electromagnetic field in relativistic-invariant form:

$$\frac{\partial W_{kl}}{\partial x_l} = \frac{4\pi}{c}\sum \langle j_k \rangle_\pm = 4\pi i \sum \int u_k \delta\left[\left(-\left(p_k - \frac{\varepsilon_\pm}{c}A_k\right)^2\right)^{1/2} - m_\pm c\right] f_\pm(x, \mathbf{p})\, d^4p, \tag{2. 5}$$

where $W_{kl} = \partial A_l/\partial x_k - \partial A_k/\partial x_l$ is the electromagnetic field tensor.

It remains to apply system (2. 1), (2. 5) to the problem posed in § 1; the solution to the system of equations of the characteristics (2. 1) yields, for each plasma component, a complete set of first integrals

$$p_2 = p_y = mcu_y + \frac{\varepsilon}{c}A = p_{y0}, \qquad p_3 = p_z = mcu_z \equiv p_{z0}, \tag{2. 6}$$

$$p_4 = \frac{i}{c}(E^\circ + \varepsilon\Phi) = \frac{i}{c}\left(c\sqrt{m^2c^2 + p_x^2 + [p_y - (\varepsilon/c)A]^2 + p_z^2} + \varepsilon\Phi\right) = p_{40}. \quad (2.6)$$

(cont'd)

Since $u_z \equiv p_{z0}/mc$, without loss of generality, we may consider all particle motion in the plane xy, setting $p_{z0} = 0$. The function $F(x, p)$ is determined for $x \to -\infty$ ($A \to 0$, $\Phi \to 0$) by assigning the distribution functions $f_\pm$ (p_{y0}, p_{40}) and relations (2. 4), (2. 6).

We will now give a particular form to Eqs. (2. 5), keeping in mind that $A_1 = A_3 = 0$, $A_2 = A$, $A_4 = i\Phi$.

$$\begin{aligned}\frac{d^2\Phi}{dx^2} = &-\frac{4\pi ie}{c^2}\left\{\frac{1}{m_+}\int E_+^\circ f_+(x, \mathbf{p})\,\delta\left(\sqrt{-[p_k - (e/c)A_k]^2} - m_+c\right)d^4p - \right.\\ &\left.-\frac{1}{m_-}\int E_-^\circ f_-(x, \mathbf{p})\,\delta\left(\sqrt{-[p_k + (e/c)A_k]^2} - m_-c\right)d^4p\right\}. \end{aligned} \quad (2.7)$$

$$\begin{aligned}\frac{d^2A}{dx^2} = &-\frac{4\pi ie}{c}\left\{\int\frac{p_y - (e/c)A}{m_+}f_+(x, \mathbf{p})\,\delta\left(\sqrt{-(p_k - (e/c)A_k)^2} - m_+c\right)d^4p - \right.\\ &\left.-\int\frac{p_y + (e/c)A}{m_-}f_-(x, \mathbf{p})\,\delta\left(\sqrt{-(p_k + (e/c)A_k)^2} - m_-c\right)d^4p\right\}. \end{aligned} \quad (2.8)$$

In the presence of a φ-function, we can discard four-dimensional integration in favor of integration with respect to the element $dp_1\,dp_2\,dp_3\,/\,E^\circ$ [12]. Transforming (2. 7), (2. 8), we obtain

$$\frac{d^2\Phi}{dx^2} = -4\pi\langle\rho\rangle = 4\pi e\left\{\int f_-(x, \mathbf{p})\,dp_x\,dp_y - \int f_+(x, \mathbf{p})\,dp_x\,dp_y\right\}, \quad (2.9)$$

$$\frac{d^2A}{dx^2} = -\frac{4\pi}{c}\langle j\rangle = 4\pi ce\left\{\int\frac{p_y + (e/c)A}{E_-^\circ}f_-(x, \mathbf{p})\,dp_x\,dp_y - \int\frac{p_y - (e/c)A}{E_+^\circ}f_+(x, \mathbf{p})\,dp_x\,dp_y\right\}. \quad (2.10)$$

System (2. 9), (2. 10) contains one first integral expressing the continuity of the T_{xx}-component of the energy-momentum tensor T_{ik} of the "particle-field" system:

$$\frac{E^2 - H^2}{8\pi} + \langle p_{xx}\rangle = \text{const}, \quad (2.11)$$

where

$$\langle p_{xx}\rangle = c^2\int\frac{p_x^2}{E_-^\circ}f_-(x, \mathbf{p})\,dp_x\,dp_y + c^2\int\frac{p_x^2}{E_+^\circ}f_+(x, \mathbf{p})\,dp_x\,dp_y. \quad (2.12)$$

The limits of integration in (2. 9)-(2. 12) are determined by the conditions

$$p_x^2 \geqslant 0, \qquad p_{x0}^2 \geqslant 0, \quad (2.13)$$

and the integrals of motion (2. 6). Inequalities (2. 13) define in momentum space the class of infinite trajectories passing through the fixed point x at which the average quantities $\langle\rho\rangle$, $\langle j\rangle$, $\langle p_{xx}\rangle$ are computed.

System of equations (2. 9), (2. 10) for the selfconsistent electromagnetic field is supplemented by the necessary four boundary conditions

$$\Phi(-\infty) = 0, \qquad A(-\infty) = 0, \quad (2.14)$$

$$E(\infty) = \Phi'(\infty) = 0, \qquad H(\infty) = A'(\infty) = H_0 = [8\pi\langle p_{xx}\rangle]^{1/2} > 0. \quad (2.15)$$

As shown in [8], conditions (2. 14), (2. 15), with the assumptions of § 1, ensure that

$$E(-\infty) = 0, \qquad H(-\infty) = 0, \quad (2.16)$$

are simultaneously satisfied.

Nevertheless, we must emphasize the fact that the problem of determining ϕ, A over the entire boundary layer is essentially a boundary problem. The attempt to solve a system similar to (2. 9), (2. 10) with conditions (2. 14), (2. 16), undertaken, for example, in [13], leads to erroneous results in view of the incorrectness of the Cauchy problem for equations of type (2. 9). In this case, the exponential growth of one of the linearly independent solutions of (2. 9) which as may easily be shown, is already contained in the general solution of the equation linerized for $x \to -\infty$ ($\Phi \to 0$, $A \to 0$), leads to instability of the numerical calculation.

The subsequent reduction of the problem to concrete form involves assigning particle distribution functions $f_\pm$ in the unperturbed plasma for $x \to -\infty$. The study that follows is based on distribution functions of the form

$$f_\pm(p_{x0}, p_{y0}) = \frac{n_0}{2|P_{1\pm}|}\delta\left(\frac{p_{y0}}{m_\pm c} - P_2\right) \quad \text{for} \quad \frac{p_{x0}^2}{m_\pm^2 c^2} \leqslant P_{1\pm}^2,$$
$$f_\pm(p_{x0}, p_{y0}) = 0 \quad \text{for} \quad \frac{p_{x0}^2}{m_\pm^2 c^2} > P_{1\pm}^2. \tag{2.17}$$

The presence of a δ-function of $(p_{y0}/m_\pm c - P_2)$ in (2.17) enables us to reduce the integrals on the right of (2.9), (2.10), and (2.12) to quadratures without detriment to the physical results. In fact, it is not difficult to see that, using a series of terms (2.17) with different P_1, P_2, taken with a suitable weight, we may formally approximate any distribution function. Naturally, in each specific physical situation the choice of distribution function itself must be determined to a significant degree by stability considerations. Note that the fact that the function (2.17) is nonlocal with respect to p_{x0} precludes the divergence of $\langle\rho\rangle$ at the reflection point, which is inevitable when monoenergetic streams are considered [1, 2]. Moreover, the distribution function chosen enables us to investigate the case of oblique incidence of the plasma on the magnetic wall ($P_2 \neq 0$).

Substituting (2.17) into (2.9), (2.10), and (2.12) and going over to the dimensionless variables

$$\psi = e\Phi/m_- c^2, \qquad a = eA/m_- c^2, \qquad \xi = x/\xi_* = (4\pi n_0 e^2/m_- c^2)^{1/2} x,$$

we obtain

$$\frac{d^2\psi}{d\xi^2} = Q_1(\psi, a) = N_-(\psi, a) - N_+(\psi, a) \quad \left(N_- = \frac{P_-}{P_{1-}},\ N_+ = \frac{P_+}{P_{1+}}\right), \tag{2.18}$$

$$\frac{d^2 a}{d\xi^2} = Q_2(\psi, a) = I_-(\psi, a) + I_+(\psi, a), \tag{2.19}$$

$$I_- = \frac{P_2 + a}{P_{1-}} \ln \frac{P_- + \sqrt{1 + P_-^2 + (P_2 + a^2)}}{\sqrt{1 + (P_2 + a)^2}},$$

$$I_+ = \frac{\mu a - P_2}{P_{1+}} \ln \frac{P_+ + \sqrt{1 + P_+^2 + (P_2 - \mu a)^2}}{\sqrt{1 + (P_2 - \mu a)^2}} \qquad \left(\mu = \frac{m_-}{m_+}\right),$$

$$P_- = \begin{cases} (P_{1-}^2 + 2\psi\sqrt{1 + P_{1-}^2 + P_2^2} + \psi^2 - 2P_2 a - a^2)^{1/2} & (P_-^2 > 0), \\ 0 \quad (P_-^2 \leqslant 0) \end{cases}$$

$$P_+ = \begin{cases} (P_{1+}^2 - 2\mu\psi\sqrt{1 + P_{1+}^2 + P_2^2} + \mu^2\psi^2 + 2\mu P_2 a - \mu^2 a^2)^{1/2} & (P_+^2 > 0). \\ 0 \quad (P_+^2 \leqslant 0) \end{cases}$$

§ 3. A solution to system (2.18), (2.19) was found by considering instead the system

$$\frac{\partial\psi^\circ(t,\xi)}{\partial t} = \frac{\partial^2\psi^\circ(t,\xi)}{\partial\xi^2} - Q_1(\psi^\circ, a^\circ), \qquad \frac{\partial a^\circ(t,\xi)}{\partial t} = \frac{\partial^2 a^\circ(t,\xi)}{\partial\xi^2} - Q_2(\psi^\circ, a^\circ), \tag{3.1}$$

whose solution, since the constant terms and the boundary conditions are independent of "time" t, embraces the steady-state regime and is linked with the solution of the initial system by the relations

$$\psi(\xi) = \lim_{t\to\infty} \psi^\circ(t, \xi), \qquad a(\xi) = \lim_{t\to\infty} a^\circ(t, \xi). \tag{3.2}$$

A difference analogue of Eqs. (3.1) was integrated on a high-speed electronic computer, using a special method that made it possible to fix the turning point of the ion from the maximum value of the x-component of the momentum; thus we were able to make a careful check on the region of maximum gradients of ψ and a.

This special approach was made necessary by the great mathematical difficulty of solving system (3.1), which was associated not only with its considerable nonlinearity but chiefly with the presence of two fundamentally different characteristic scales of variation of the functions for each of Eqs. (3.1).

In fact, in a number of cases the spatial scale of variation of the magnetic field R (ion Larmor radius) differs sharply from the Debye radius D – the characteristic length at which significant charge separation is to be expected (Fig. 2).

In the following section some results are given for calculation of specific physical examples according to the above method.

§ 4. Figure 2 gave a typical picture of the space distribution of the principal physical quantities in selfconsistent electrostatic and magnetic fields corresponding to a plasma with parameters $\mu = 1/1836$, $P_{1-} = 10^{-1}$, $P_{1+} = 2.24 \times 10^{-3}$, $P_2 = 0$, where in accordance with § 2,

$$P_{1\pm} = \frac{V_{1\pm}}{c}\left[1-\left(\frac{V_{1\pm}}{c}\right)^2\right]^{-1/2}.$$

Here and in Figs. 3, 6, 7, the magnitudes of the fields H and E (curves 1 and 2) are shown by heavy continuous lines, the total current density (curve 1) and the charge density ρ (curve 2) by thin continuous lines. The dot-dash curves and 2 correspond to average values of the transverse velocity of the negative (v_-) and positive (v_+) components. The broken-line curve in Fig. 3 corresponds to the current density j_-, while in Fig. 7 it corresponds to the charge density ρ_-.

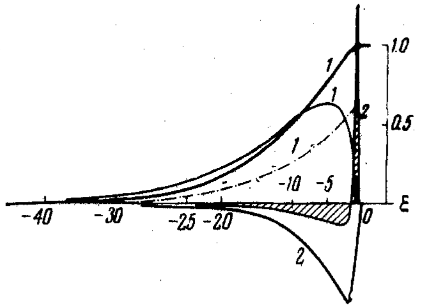

Fig. 2

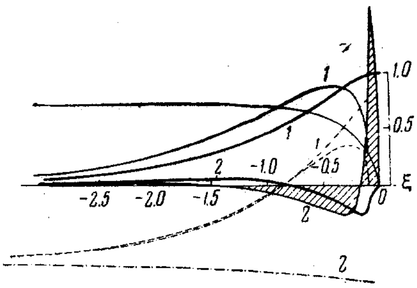

Fig. 3

Figure 2 shows a typical spatial picture of the boundary layer with:

$$[E] = 0.025\frac{m_-c^2}{e\xi_*}E,\ [H] = 0.1157\frac{m_-c^2}{e\xi_*}H,\ [j] = 10^{-2}ecn_0j,\ [\rho] = 10^{-2}en_0\rho,\ [v_-] = 0.02cv_-.$$

(here and below dimensionless quantities are contained in square brackets).

Figure 3 shows the boundary layer for oblique incidence of the plasma on the "magnetic wall" with:

$$\mu = 0.1,\quad P_{1+} = P_{1-} = 0.1,\quad P_2 = P_{2*} = -0.142,$$

$$[E] = 0.1\frac{m_-c^2}{e\xi_*}E,\quad [H] = 0.269\frac{m_-c^2}{e\xi_*}H,\quad [j] = 0.2ecn_0j,$$

$$[j_+] = 0.2ecn_0j_+,\ [j_-] = 0.2ecn_0j_-,\ [\rho] = 0.2en_0\rho,\ [v_-] = 0.2cv_-,\ [v_+] = 0.2cv_+.$$

In both figures the distance is plotted along the x-axis in units of

$$\xi_* = \left(\frac{m_-c^2}{4\pi n_0e^2}\right)^{1/2}.$$

At nonrelativistic plasma particle energies, ξ_* is of the order of $\sqrt{Rr}$, where R and r are, respectively, the particle Larmor radii with $p_+ = m_+cP_{1+}$, $p_- = m_-cP_{1-}$. These particles, possessing (cf. (2.17)) a maximum initial momentum in a direction normal to H_0, will be called probe particles.

The characteristic flash of positive charge and, in accordance with [1], the almost complete energy exchange between the heavy and light plasma components at the reflection point of the probe electron ($v_- \gg v_+$) are shown quite clearly in Fig. 2. The magnetic field strength decreases by an order of magnitude at a distance of roughly $24\xi_*$; in this case

$$R = 38.7\xi_*,\ \sqrt{Rr} = 5.785\xi_*.$$

An interesting situation arises upon oblique incidence of the particles on the magnetic field. When $P_2 < 0$, the electrons, which lead the ions in the initial stage of motion, acquire the possibility of creating an additional excess positive charge density. Instead of a double, we get a triple charged layer. At some $P_2 = P_2 < 0$, the jump in electrostatic potential $\delta\psi = \psi(\infty)$, and with it the dipole moment of the boundary layer, vanish (Fig. 3), after which, with further decrease in P_2, $\delta\psi$ changes sign. At $P_{1+} = P_1 = P_1$, P_2 is determined by the simple formula

$$P_{2*} \approx -\frac{P_1(1-\mu)}{2\sqrt{\mu}} \tag{4.1}$$

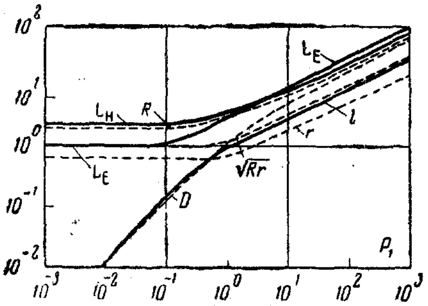

Fig. 4. Characteristic dimensions of boundary layer as a function of particle momentum in the plasma (double logarithmic scale).

As calculations show, a decrease in μ, with all other plasma parameters remaining the same, leads to a sharp increase in the amplitudes of ρ, E, and other quantitative characteristics (in this connection the machine time also increases), the qualitative effect being much less significant. It is therefore of interest to perform a series of calculations at an intermediate value of ρ, but over a wide range of energies, so as to trace the nature of the changes in the structure of the boundary layer upon transition from the nonrelativistic to the ultrarelativistic region.

The corresponding data are given in Figs. 4, 5. Here $\mu = 0.25$, $P_2 = 0$, $P_{1+} = P_{1-} = P_1$ and varies from 10^{-3} to 10^3. All the linear dimensions in Fig. 4 are given in units of ξ_* and are measured from the origin, which is coincident with the point of reflection of the probe ion, while, by definition of L_H, L_E and l,

$$H(-L_H) = 0.1H_0, \qquad E(-L_E) = 0.1E_{max},$$
$$\rho(-l) = 0.$$

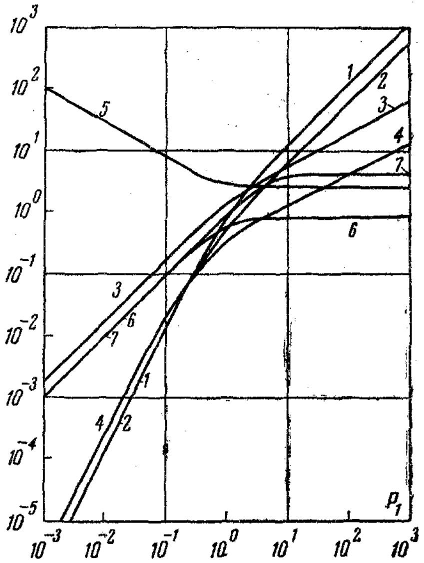

Fig. 5. Principal physical quantities as a function of the momentum of particles incident on a "magnetic wall."

(1) $\psi^\circ = \frac{e[\Phi^\circ]}{m_- c^2}$, (2) $\psi_{max} = \frac{e[\Phi_{max}]}{m_- c^2}$

(3) $H_0 = \frac{e\xi_+}{m_- c^2} H_0$

(4) $(-E_{max}) = \frac{e\xi_+[-E_{max}]}{m_- c^2}$

(5) $\frac{\rho_+}{\rho_-}$, (6) $j_- = \frac{[j_-]}{ecn_0}$, (7) $j = \frac{[j]}{ecn_0}$.

The quantity D, which serves as Debye radius, is related with the kinetic energy of the probe particles as follows:

$$D^2 = \mu^{-1}[(1 + P_{1+}^2 + P_2^2)^{1/2} - 1] + (1 + P_{1-}^2 + P_2^2)^{1/2} - 1.$$

The notations $H_0, E_{max}, \rho_+, \rho_-, j_-, j$ and $\psi_{max} = \delta\psi$ in Fig. 5 denote maximum values of the corresponding quantities, and ψ° denotes the value of the electrostatic potential at which the probe particles would be reflected at one point.

It is interesting to note that (as follows directly from Figs. 4, 5) the dependence upon particle momentum, for the majority of quantities determining the structure of the boundary layer, is exponential in nature in the nonrelativistic and ultrarelativistic regions. Data given below illustrate the energy redistribution between components at the probe particle reflection points.

$P_1 =$	10^{-3}	10^{-2}	10^{-1}	1	10	10^2	10^3
$\frac{\max v_-}{c} =$	$0.19998 \cdot 10^{-2}$	$0.19958 \cdot 10^{-1}$	0.18986	0.85654	0.99329	0.99954	0.99998
$\frac{\max v_+}{c} =$	$0.50006 \cdot 10^{-3}$	$0.50132 \cdot 10^{-2}$	$0.52756 \cdot 10^{-1}$	0.58885	0.99065	0.99877	0.99969

We recall that, according to the "neutral" layer theory, $\max v_- / \max v_+ = 4$ at $\mu = 0.25$. With increase in energy this ratio changes and finally approaches unity, while v- and v approach the speed of light c.

Figures 6 and 7 both for ($\mu = 0.25$) show the structure of the boundary layer for relatively low ($P_1 = 10^{-2}$) and relativistic ($P_1 = 10$) energies.

Naturally, it is impossible to encompass the every problem associated with the theory of the stationary boundary layer in one article. Our real aim is to draw attention to the possibilities of the method used and to note the laws following from a consideration of the complete spatial picture of a selfconsistent electromagnetic field in a boundary layer.

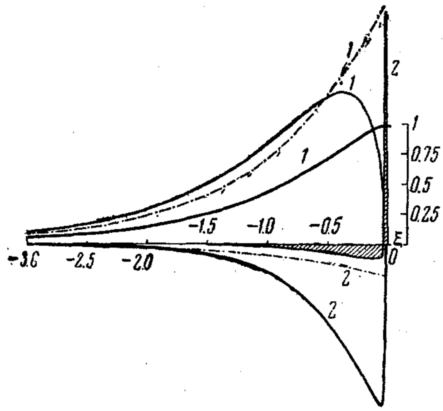

Fig. 6. Boundary layer for nonrelativistic plasma particle energies.

$$[E] = 10^{-4}\frac{m_- c^2}{e\xi_*}E$$

$$[H] = 0.1826\cdot 10^{-1}\frac{m_- c^2}{e\xi_*}H$$

$$[j] = 2\cdot 10^{-3} ecn_0 j, \quad [\rho] = 4\cdot 10^{-2} en_0\rho$$

$$[v_-] = 10^{-2} cv_-, \quad [v_+] = 2\cdot 10^{-2} cv_+$$

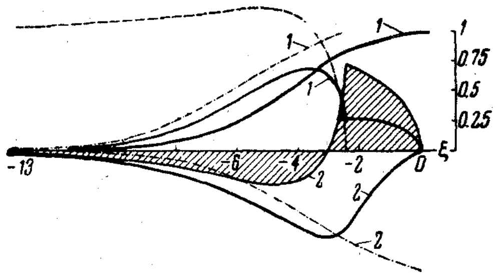

Fig. 7. Boundary layer between magnetic field and relativistic plasma

$$[E] = 20\frac{m_- c^2}{e\xi_*}E$$

$$[H] = 5\cdot 323\frac{m_- c^2}{e\xi_*}H$$

$$[j_-] = 2ecn_0 j_-, \quad [j] = 2ecn_0 j,$$

$$[\rho] = en_0\rho, \ [v_-] = cv_-, \quad [v_+] = cv_+$$

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